Final Exam

Name:

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8	/10

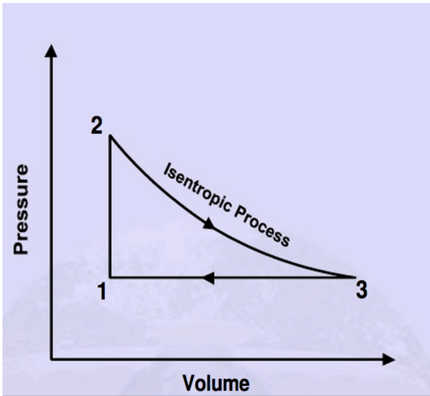
1. Carnot cycle: Two identical bodies with temperature independent heat capacities C are initially at different temperatures T_H and T_C. A Carnot cycle is run between them (with infinitesimal steps) until they have been reduced to a common temperature T_F. [15 points]

a) Does the efficiency of this engine change over time? [5 points]

b) In a Carnot cycle, the entropy change is zero per cycle as we saw in the previous exam. Find T_F in terms of T_H and T_C. The answer is not $(T_H+T_C)/2$. Consider the entropy change in the hot and cold reservoir: these have to be equal. Integrate from initial temperatures to find the solution. [5 points]

c) Find the total work done on the outside world in this process. Is it positive or negative? [5 points]

2. Consider a Lenoir cycle as given by the figure below. We note that efficiency can be given by (Heat added)-(Heat rejected)/Heat added. Relevant temperatures are T_1, T_2, T_3. (Isentropic process = reversible adiabatic process). [15 points]



(a) Calculate T_3 in terms of T_2 [5 points]

(b) When is heat being added? [2.5 points]

(c) When is heat being rejected? [2.5 points]

(d) Calculate efficiency in terms of T_1, T_2, and T_3. as well as Nk (assume monoatomic gas is involved) [5 points] [Hint: C_v=3/2 Nk, C_p=5/2Nk, dU+pdV = $\Delta H = C_p \Delta T$]

3. Manipulation of thermodynamic quantities [15 points]

In a weakly interacting gas of Bose particles at low temperature the expansion coefficient α and the isothermal compressibility $\mathcal{K}_T$ are given by

$$\alpha \equiv \frac{1}{V}\frac{\partial V}{\partial T}\bigg|_P = \frac{5}{4}\frac{a}{c}T^{3/2}V^2 + \frac{3}{2}\frac{b}{c}T^2V^2$$

$$\mathcal{K}_T \equiv -\frac{1}{V}\frac{\partial V}{\partial P}\bigg|_T = \frac{1}{2c}V^2$$

where a, b and c are constants. It is known that the pressure goes to zero in the limit of large volume and low temperature. Find the equation of state $P(T, V)$.

Hint: You may use that (dP/dT) at constant volume $=\alpha/\kappa_T$

4. Consider a simple harmonic oscillator with energy given by $E = n\hbar\omega$, where ω is a constant given by sqrt (k/m). [15 points]

(a) Calculate its partition function. You may utilize this formula below to simplify your infinite sum. $\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$ [3 points]

(b) What is its energy at some temperature T? [3 points]

(c) What is the meaning of <n>? (Average n) It is not an occupation number. [3 points]

(d) Calculate heat capacity of this simple harmonic oscillator. [3 points]

(e) Calculate heat capacity at high temperature ($kT >> \hbar\omega$) [3 points]

5. If a partition function for a single particle is given by Z, find out the partition function for N particles [15 points]

(a) provided that they are distinguishable particles [7.5 points]

(b) provided that they are indistinguishable particles [7.5 points]

6. Consider a system composed of N spins in which energy levels are defined to be $-\mu B$, 0, μB and magnetic moment is for these levels are given by μ, 0, $-\mu$.

(a) What is the partition function? [3 points]

(b) What is the probability of finding a spin with magnetic moment of 0 at infinite temperature? [3 points]

(c) What is the average energy for this system? [3 points]

(d) What is the Helmholtz free energy of this system? [3 points]

(e) What is the entropy of the system? [3 points]

7. Consider a system consisting of a single hydrogen atom/ion, which has two possible states: unoccupied with no electrons and occupied with one electron present. i.e. unoccupied state has zero energy with zero electrons and occupied state has energy -I and has one electron in it with a chemical potential of μ. Calculate the ratio of the probability of these two states. You may use $\mu=-kT \ln(V/NV_Q)$ to simplify the formula. Simplify such that there is only one exponential in the final ratio. You can assume that electrons behave like an ideal gas ($PV=NkT$). Use only P, V_Q, k, T, I to express the ratio. [15 points]

8. Calculate the densities of states for a 1D electron gas given that the allowed k vectors are $k=(2\pi/L)*m$ with m being all integers. Hints: 1. calculate the density of states in therms of k, 2. use the energy formula $(\hbar k)^2/2m$, 3. use $N = 2* 2k*D(k)$ [10 points]